Written Exam for the M.Sc. in Economics Summer 2012

Fixed Income Derivatives — Solution

Introduction

The purpose of the exam is to test the student's ability to price and risk manage derivatives. In order to receive the grade 12, the student must demonstrate the ability to implement and use the models and methodologies covered in the course.

1 The Swap Curve

1.1 Introducing money market basis

In the traditional single curve setup, any xIBOR rate can be seen as a discounting weighted average of any set of sub-periods that spans the tenor of the specific xIBOR rate. In particular, a 6M xIBOR rate with spot start could be viewed as being spanned by a 3M rate with spot start and the subsequent 3M forward xIBOR (fixing in 3M time). Disregarding discounting, it easily seen that the average rate implied over the full 6M period by the two 3M EURIBOR based instruments is $(0.25*0.753\%+0.25*0.923\%)/0.5 = 0.838\%$. This compares to a 6M EURIBOR fixing of 1.048%. This difference of 21 bps is the added credit premium embedded in the 6M EURIBOR fixing compared to the 6M average rate implied by the two 3M EURIBOR rates.

1.2 Pricing money market basis swaps in VBA

See the VBA code in the solution spreadsheet. Naturally, there is more than one way of implementing the code. Students should recognize that in order to solve the problem, one simply needs to compute the value of two floating legs — each valued on its own forward curve while using the same discounting curve — and divide the difference by the annuity value.

1.3 Calibrating a swap market with multiple forward curves

The calibrated curves can be seen in table 1 below. A margin of error of ± 2 bps is allowed in order to account for differences in calibration starting points. The graphs for the relevant forward xIBOR rates can be seen in figure 1 and figure 2.

Table 1: Calibrated zero coupon curves (continuously compounded, Act/365)

Maturity	Date	Fwd rates, 3M	Fwd rates, 6M	Disc rates
1Y	23-04-2013	0.6583%	0.9498%	0.0799%
2Y	23-04-2014	0.7552%	1.0163%	0.2109%
5Y	24-04-2017	1.2650%	1.4643%	0.8158%
10Y	25-04-2022	2.0638%	2.2089%	1.7930%
15Y	23-04-2027	2.7400%	2.8552%	2.6084%
30Y	23-04-2042	3.1784%	3.2392%	3.1347%

Figure 1: Forward xIBOR curves.

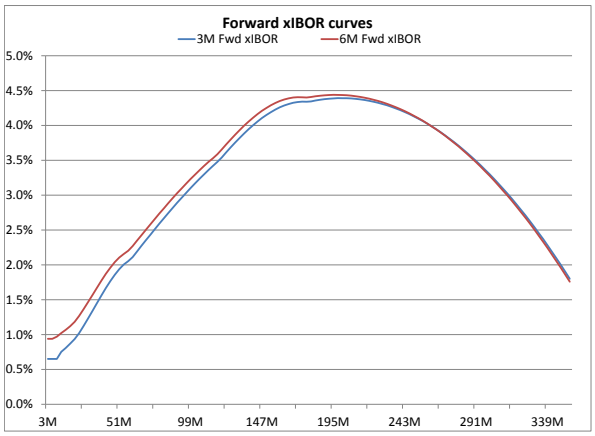
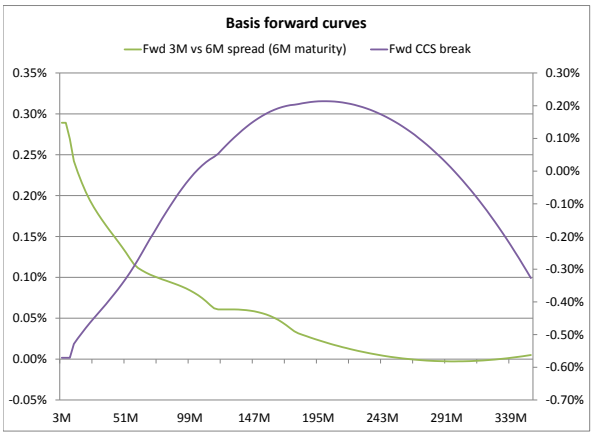


Figure 2: Forward basis spreads.



1.4 Terminating an IRS

On the full set of calibrated curves, the swap has a mid-market PV of EUR -8,923,021. Given the uncertainty of the curve calibration, any PV within EUR ± 170,000 should be accepted (approximately 2 bps of the swap's net delta risk). Students should recognize that since the swap fixes against a floating rate tenor of 3M, it is the 3M forward curve that should be used to compute forward rates.

Being ignorant about both money market and cross currency basis would lead to the calibration (which then would consist of just a single zero coupon curve) seen in table 2. On this curve, the mid-market PV of the swap would be EUR -7,609,583. Due to the simpler curve calibration problem, the accepted error should here be lower at EUR ± 90,000.

Table 2: Calibrated zero coupon curve – single curve setup (continuously compounded, Act/365)

Maturity	Date	Single curve rates
1Y	23-04-2013	0.947%
2Y	23-04-2014	1.015%
5Y	24-04-2017	1.466%
10Y	25-04-2022	2.218%
15Y	23-04-2027	2.866%
30Y	23-04-2042	3.243%

There are two effects that causes the swap's PV to be different between the full- and the single curve setup. Firstly, if the swap is valued on the single curve setup — which is calibrated to 6M EURIBOR indexed swaps — the projected floating rate cash flows will be too high. Since we are paying the fixed rate in the swap, this will cause us to overestimate the value of the floating rates that we are receiving and thus underestimate the negative value of the swap. Secondly, the presence of the cross currency basis causes us to set a discounting curve that is substantially below the the discounting curve we would use in the single curve setup. This means that we in the single curve setup will discount the negative cash flows that the high fixed rate entails harder — again this will cause us underestimate the negative value of the swap.

1.5 Calculating market rate delta vectors in the presence of money market basis

The swap's market rate delta vector can be seen in table 3. Intuitively, the swap positioned for the general level of interest rates to rise (this is sometimes called the outright level), since we are receiving floating rates. Since the IRS calibration instruments are indexed against 6M EURIBOR and we are receiving 3M EURIBOR rates in the swap, we are at the same time positioned for a decrease in the 3M-6M basis spread as indicated by the negative money market basis sensitivity. A decrease in this basis spread will — for a given level of 6M forward rates — cause 3M forward rates to rise. Lastly, an increase in the cross currency basis spreads (i.e. a move towards zero from the current negative levels) effectively lift the discounting curve. Since we are net paying out cash over most of the swap's life (the fixed rate is higher than the 3M forward rate), we will benefit from such an increase in discounting rates.

Table 3: Market rate delta vector, EUR per bps.

Maturity	IRS	CCS	MM Basis
1Y	-2,752	-80	2,791
2Y	395	321	-400
5Y	-3,222	2,130	3,279
10Y	54,280	2,493	-55,439
15Y	39,281	759	-40,226
30Y	-1,025	-8	1,052
Total	86,956	5,616	-88,943

1.6 Money market basis as two swaps

By paying the fixed rate in, say, an IRS indexed against 3M EURIBOR while receiving the fixed rate in an IRS indexed against 6M EURIBOR, we would effectively be receiving 3M EURIBOR while paying 6M EURIBOR as the two fixed cancel out. Intuitively, the sensitivity towards the overall level of interest rates is quite limited in such a two-swap trading strategy — the outright risk simply cancels out. The combined position will however be sensitive towards an isolated in the 3M forward relative to the 6M forward curve or vice versa.

When computing basis spreads in two-swap version compared to the one-swap version, the value of the two floating legs are naturally the same. The difference lies in the way the

money market basis spread is paid. Firstly, in the one-swap version, the spread is paid as a quarterly, Act/360 spread while it is paid as an annual, 30/360 spread in the two-swap version. Since the Act/360 convention generates higher higher coverages (on average 365.25/360>1 per year) compared to the 30/360 convention (on average 360/360=1 per year), the spread in the former must be slightly lower in order to generate the same net cash flow. Secondly, cash flows are received earlier (every quarter) in the one-swap version compared to the two-swap version (every year). This effect also means that the spread must be a little higher in the two-swap version to compensate for the discounting effect.

The basis spreads in the two versions can be seen in the table 4. Errors of ± 1 bps should be accepted.

Table 4: Money market basis spreads — one or two swaps.

Start	Maturity	Par spread (float-float, 1 swap)	Par spread (fixed-float, 2 swaps)
2B	1Y	28.91	29.33
2B	2Y	25.90	26.28
2B	5Y	19.87	20.22
2B	10Y	14.75	15.07
2B	15Y	12.12	12.41
2B	30Y	8.02	8.22

2 The EUR cap/floor market

2.1 Calibrating a grid of SABR models

Since the calibration involves fitting three SABR models to the entire set of caplets (19 in total per strike), the system is somewhat underdetermined — even if the smile structure on each caplet could be adequately described by the SABR model functional form. Unintentionally, this means that multiple calibrations leads to fits that are roughly of the same quality. As each of such calibrations has different combinations of parameters, it will not make sense to judge students on the fit of the parameters alone. Rather, the fit should be evaluated on the pricing of the below floor. As a reference point for the below pricing, the SABR calibration in table 5 can be used.

Table 5: One plausible SABR calibration — reference only.

Maturity	Date	σ_0	ϵ	ρ
2Y	22-04-2014	6.37%	72.46%	-81.31%
5Y	24-04-2017	7.30%	41.08%	-27.77%
10Y	22-04-2022	0.01%	49.69%	2.83%

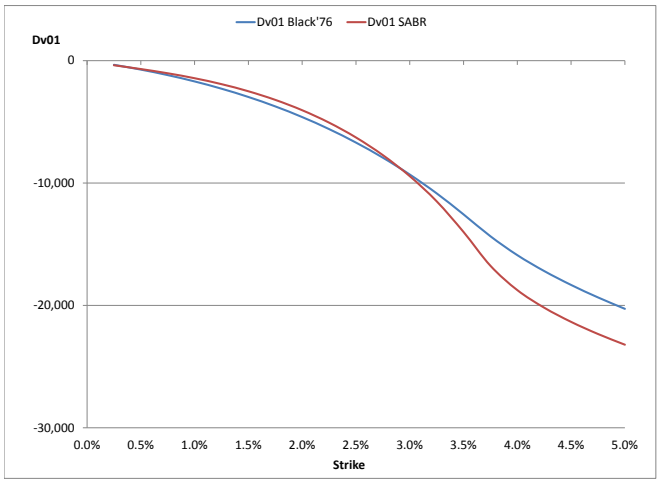
2.2 Pricing a forward starting floor

On the above calibration, the forward starting floor has a PV of EUR 752,182. Due to the issues around calibration, any price wihtin EUR ± 100,000 (or approx 3 times the vega of the floor contract) should be fully accepted.

2.3 SABR vs. Black'76

The floor has a net Dv01 of EUR -3,744 in the Black'76 model compared to a net Dv01 of EUR -3,204 in the SABR model using the SABR calibration seen in table 5. Naturally, different SABR calibrations will provide different Dv01 numbers. A acceptable margin of error on the Dv01 numbers is EUR ± 500. Students should be able to reproduce a graph qualitatively similar to figure 3.

Figure 3: EUR 5X10 Floor, EUR 50m notional



In the Black'76 model, the implied volatility (in log-normal terms) stays unchanged when the underlying forward rate moves. This violates two empirical findings. Firstly, many markets exhibit a co-movement between the underlying and the ATM log-normal volatility such that the log-normal volatility decreases when the underlying increases. Also, the volatility smile moves with the underlyings. That this, as the relative moneyness of an option changes when the underlying moves. This can cause volatility to both in- and decrease. It is this co-movement between implied volatility and the underlying forward rate that causes the Dv01 to differ between the two models.

3 The CDS market

3.1 Calibrating a CDS curve

The calibrated intensities can be seen in the table 6. An error of ± 1 bps should accepted.

Table 6: Calibrated intensities.

Maturity	Date	Intensity curve
2Y	22-Apr-14	2.8704%
5Y	24-Apr-17	2.7083%
10Y	22-Apr-22	2.2143%

3.2 Price a forward starting CDS

On the calibrated curve, the par CDS spread is 1.314% (131.4 bps). An error of ± 1 bps should be accepted due to the calibration uncertainty in the question above.

3.3 The risk profile of a forward starting CDS

Intuitively, we can think of a CDS where we have bought protection as being positioned for an increase in the likelihood of default over the period during which we have bought protection. However, since default can only happen once, the payout of the protection leg is conditional on no prior default.

As intensities increase, the likelihood of default obviously increases for the spot starting swap. For the forward starting swap however, high levels of the default intensities makes default prior to the swap's start so likely that the likelihood that default occurs during its life actually decreases. This is what causes the PV graph in figure 4 to become downward sloping beyond the 15% point. In the limit, the underlying name will default with certainty before the start of swap and the forward starting CDS will be worthless.

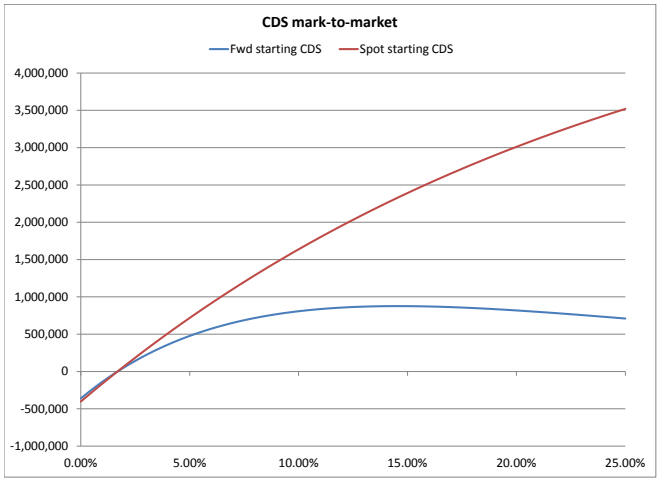



Figure 4: PV of EUR 10m CDS at varying intensities.